

# Baryogenesis in the Zee-Babu model

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Received: Received: date / Revised version: date

**Abstract.** We consider the baryogenesis picture in the Zee-Babu model. Our analysis shows that electroweak phase transition (EWPT) in the model is a first-order phase transitions at the 100 GeV scale, its strength is about  $1 - 2.4$  and the masses of charged Higgs are smaller than 350 GeV. The EWPT is strengthened by only the new bosons. The process of the EWPT kinetics corresponding to B violation which is received through the sphaleron probability. By a thin-wall approximation, we estimate that sphaleron rate is larger than the cosmological expansion rate at the temperature which is higher than the critical temperature, and after the phase transition, the sphaleron process is decoupled. This also suggests that the phase transition is a transition which depends at each point in space. This may provide baryon-number violation (B-violation) necessary for baryogenesis in the relationship with non-equilibrium physics in the early universe.

**PACS.** 11.15.Ex Spontaneous breaking of gauge symmetries, 12.60.Fr Extensions of electroweak Higgs sector – 98.80.Cq Particle-theory models (Early Universe)

arXiv:1511.00579v3 [hep-ph] 23 Mar 2016

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Keywords: Spontaneous breaking of gauge symmetries, Extensions of electroweak Higgs sector, Particle-theory models (Early Universe)

## 1 INTRODUCTION

Physics, at present, has entered into a new period, understanding of the early Universe. In that context, almost Cosmology and Particle Physics are on the same way. In the current issue, the baryon asymmetry is an interesting problem, it is seen as a central issue of cosmology and particle physics. If we explain this problem, we can understand the true nature of the smallest elements and reveal a lot about a imbalances matter-antimatter from the early Universe.

The electroweak baryogenesis (EWBG) is a way to explaining the Baryon Asymmetry of Universe (BAU) in the early universe, has been known by Sakharov conditions, which are B, C, CP violations, and deviation from thermal equilibrium [1]. These conditions can be satisfied when the EWPT must be a strongly first-order phase transition. Because that not only leads to thermal imbalance [2], but also makes a connection between B and CP violation via nonequilibrium physics [3].

The EWPT has been investigated in the Standard Model (SM) [2,4,5] as well as various extension models [6,7,8,9,10,11,12,14,15,16]. For the SM, although the EWPT strength is larger than unity at the electroweak scale, but the mass of the Higgs boson must be less than 125 GeV [2,4,5]; so the EWBG requires new physics beyond the SM at the weak scale [6].

Many extensions such as the Two-Higgs-Doublet model, the reduced minimal 3-3-1 model, the economical 3-3-1 model or the Minimal Supersymmetric Standard Model, have a strongly first-order EWPT and the new sources of CP violation, which are necessary to account for BAU; triggers for the first-order EWPT in these models are heavy bosons or dark matter candidates [7,8,9,10,11].

The quantity of sphaleron rate which is B violation rate, has been calculated in SM [2,4,5] and the reduced minimal 3-3-1 model [11]. In addition, by using non-perturbative lattice simulations, a powerful framework and set of analytic and numerical tools have been developed by authors in Ref. [4,5].

The Zee-Babu (ZB) model is the simplest extension of SM which has some interesting features [13]. We have considered the EWPT and sphaleron rate in the the ZB model due to its simplicity.

The ZB model has more two charged scalars  $h^\pm$  and  $k^{\pm\pm}$  in the Higgs potential. The kind of new scalars can play important role in the early universe. They can be a reason for tiny mass of neutrino through two loops or three loops corrections [13,17]. One important property of them which will be shown in this paper, can be triggers for the first-order phase transition.

This paper is organized as follows. In Sect. 2 we give a short review of the ZB model and we drive an effective potential which has a contribution from heavy scalars at one-loop level. In Sect. 3, we find the range mass of charged scalar particles by a first-order phase transition condition. In Sect. 4, We offer the solutions of VEV and estimate sphaleron rate by our approximations, and show that this rate can satisfy the decoupling condition. Finally, constraints on the mass of the charged Higgs boson are derived in Sect. 5. In Sect. 6 we summarize and describe outlooks.

## 2 EFFECTIVE POTENTIAL IN THE ZEE-BABU MODEL

The Lagrangian in Zee-Babu model is the Lagrangian of SM when adding two charged scalar fields,  $h^\pm$  and  $k^{\pm\pm}$  [13],

$$\mathcal{L} = L_{SM} + f_{ab}\overline{\psi}_{aL}^c\psi_{bL}h^+ + h'_{ab}\overline{l}_{aR}^c l_{bR}k^{++} + V(\phi, h, k) + (D_\mu h^+)^\dagger(D^\mu h^+) + (D_\mu k^{++})^\dagger(D^\mu k^{++}) + H.c \quad (1)$$

The Higgs potential [13] in this model has four couplings among  $h^\pm$  or  $k^{\pm\pm}$  with neutral Higgs:

$$V = m_H'^2\phi^\dagger\phi + m_h'^2|h|^2 + m_k'^2|k|^2 + \lambda_H(\phi^\dagger\phi)^2 + \lambda_h|h|^4 + \lambda_k|k|^4 + \lambda_{hk}|h|^2|k|^2 + \lambda_{hH}|h|^2\phi^\dagger\phi + \lambda_{kH}|k|^2\phi^\dagger\phi + (\mu h^2 k^{++} + H.c) \quad (2)$$

where

$$\phi = \begin{pmatrix} \rho^+ \\ \rho^0 \end{pmatrix} \quad (3)$$

and  $\rho^0$  has a VEV

$$\rho^0 = \frac{1}{\sqrt{2}}(v_\rho + \sigma + i\zeta) . \quad (4)$$

From (1), we can obtain an effective potential with contributions of  $h^\pm$  and  $k^{\pm\pm}$ :

$$\begin{aligned}
V_{eff}(v_\rho) = & V_0(v_\rho^2) + \frac{3}{64\pi^2} \left( m_Z^4(v_\rho) \ln \frac{m_Z^2(v_\rho^2)}{Q^2} + 2m_W^4(v_\rho) \ln \frac{m_W^2(v_\rho^2)}{Q^2} - m_t^4(v_\rho) \ln \frac{m_t^2(v_\rho^2)}{Q^2} \right) \\
& + \frac{1}{64\pi^2} \left( 2m_{h^\pm}^4(v_\rho) \ln \frac{m_{h^\pm}^2(v_\rho^2)}{Q^2} + 2m_{k^{\pm\pm}}^4(v_\rho) \ln \frac{m_{k^{\pm\pm}}^2(v_\rho^2)}{Q^2} + m_H^4(v_\rho) \ln \frac{m_H^2(v_\rho^2)}{Q^2} \right) \\
& + \frac{3T^4}{4\pi^2} \left\{ F_-\left(\frac{m_Z(v_\rho^2)}{T}\right) + F_-\left(\frac{m_W(v_\rho^2)}{T}\right) + 4F_+\left(\frac{m_t(v_\rho^2)}{T}\right) \right\} \\
& + \frac{T^4}{4\pi^2} \left\{ 2F_-\left(\frac{m_{h^\pm}(v_\rho^2)}{T}\right) + 2F_-\left(\frac{m_{k^{\pm\pm}}(v_\rho^2)}{T}\right) + F_-\left(\frac{m_H(v_\rho^2)}{T}\right) \right\}
\end{aligned}$$

with  $v_\rho$  is a variable which changes with temperature; at  $0^\circ K$ ,  $v_\rho \equiv v_0 = 246$  GeV and

$$\begin{aligned}
F_\pm\left(\frac{m_\phi}{T}\right) &= \int_0^{\frac{m_\phi}{T}} \alpha J_\mp^1(\alpha, 0) d\alpha, \\
J_\mp^1(\alpha, 0) &= 2 \int_\alpha^\infty \frac{(x^2 - \alpha^2)^{\frac{1}{2}}}{e^x \mp 1} dx,
\end{aligned}$$

and putting

$$\begin{aligned}
M_{h^\pm}^2 &= \frac{\lambda_{hH} v_\rho^2}{2}, \\
M_{k^{\pm\pm}}^2 &= \frac{\lambda_{kH} v_\rho^2}{2}.
\end{aligned}$$

However, the masses of  $h^\pm$  and  $k^{\pm\pm}$  are  $M_{h^\pm}$  and  $M_{k^{\pm\pm}}$ :

$$\begin{aligned}
m_{h^\pm}^2 &= m_h'^2 + M_{h^\pm}^2, \\
m_{\pm\pm}^2 &= m_k'^2 + M_{k^{\pm\pm}}^2.
\end{aligned}$$

### 3 ELECTROWEAK PHASE TRANSITION IN THE ZEE-BABU MODEL

We write the high-temperature expansion of the potential (5) as a quartic equation in  $v_\rho$ :

$$V_{eff}(v) = D(T^2 - T_0^2)v_\rho^2 - ET|v_\rho|^3 + \frac{\lambda_T}{4}v_\rho^4, \quad (5)$$

in which

$$\begin{aligned}
D &= \frac{1}{24v_0^2} [6m_W^2(v_0) + 3m_{Z_1}^2(v_0) + m_H^2(v_0) + m_{h^\pm}(v_0) + 2m_{k^{\pm\pm}}^2(v_0) + 6m_t^2(v_0)], \\
T_0^2 &= \frac{1}{D} \left\{ \frac{m_H^2(v_0)}{4} - \frac{1}{32\pi^2 v_0^2} (6m_W^4(v_0) + 3m_{Z_1}^4(v_0) + m_H^4(v_0) \right. \\
&\quad \left. + m_{h^\pm}^4(v_0) + 2m_{k^{\pm\pm}}^4(v_0) - 12m_t^4(v_0)) \right\}, \\
E &= \frac{1}{12\pi v_0^3} (6m_W^3(v_0) + 3m_{Z_1}^3(v_0) + m_H^3(v_0) + m_{h^\pm}^3(v_0) + 2m_{k^{\pm\pm}}^3(v_0)), \\
\lambda_T &= \frac{m_H^2(v_0)}{2v_0^2} \left\{ 1 - \frac{1}{8\pi^2 v_0^2 (m_H^2(v_0))} \left[ 6m_W^4(v_0) \ln \frac{m_W^2(v_0)}{bT^2} \right. \right. \\
&\quad \left. + 3m_{Z_1}^4(v_0) \ln \frac{m_{Z_1}^2(v_0)}{bT^2} + m_{H^0}^4(v_0) \ln \frac{m_{H^0}^2(v_0)}{bT^2} \right. \\
&\quad \left. \left. + 2m_{k^{\pm\pm}}^4(v_0) \ln \frac{m_{k^{\pm\pm}}^2(v_0)}{bT^2} - 12m_t^4(v_0) \ln \frac{m_t^2(v_0)}{b_F T^2} \right] \right\}, \quad (6)
\end{aligned}$$

where  $v_0$  is the value at which the zero-temperature effective potential  $V_{eff}^{0^{\circ}K}(v)$  gets the minimum. Here, we acquire  $V_{eff}^{0^{\circ}K}$  from  $V_{eff}$  in Eq. (5) by neglecting all terms in the form  $F_{\mp}(\frac{m}{T})$ .

The minimum conditions for  $V_{eff}^{0^{\circ}K}(v)$  are

$$V_{eff}^{0^{\circ}K}(v_0) = 0, \quad \left. \frac{\partial V_{eff}^{0^{\circ}K}(v)}{\partial v} \right|_{v=v_0} = 0, \quad \left. \frac{\partial^2 V_{eff}^{0^{\circ}K}(v)}{\partial v^2} \right|_{v=v_0} = [m_H^2(v_0)] \Big|_{v=v_0}. \quad (7)$$

We also have the minima of the effective potential (5):

$$v = 0, \quad v \equiv v_c = \frac{2ET_c}{\lambda_{T_c}}, \quad (8)$$

where  $v_c$  is the critical VEV of  $\phi$  at the broken state, and  $T_c$  is the critical temperature of phase transition which is given by

$$T_c = \frac{T_0}{\sqrt{1 - E^2/D\lambda_{T_c}}}. \quad (9)$$

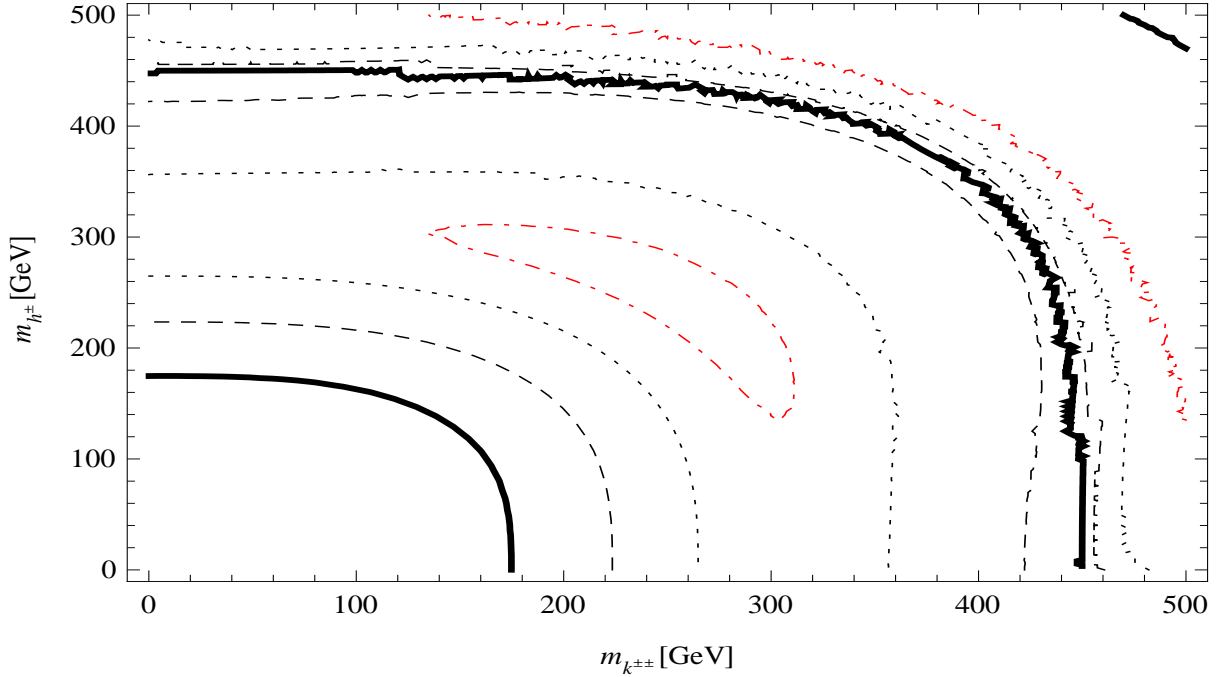
We investigate the phase transition strength

$$S = \frac{v_{\rho_c}}{T_c} = \frac{2E}{\lambda_{T_c}} \quad (10)$$

of this EWPT. In the limit  $E \rightarrow 0$ , the transition strength  $S \rightarrow 0$  and the phase transition is a second-order. To have a first-order phase transition, we requires  $S \geq 1$ . We plot  $S$  as a function of  $m_{h^{\pm}}$  and  $m_{k^{\pm\pm}}$  in Fig. 1.

According to Ref. [24], the accuracy of a high-temperature expansion for the effective potential such as that in Eq. (5) will be better than 5% if  $\frac{m_{boson}}{T} < 2.2$ , where  $m_{boson}$  is the relevant boson mass. Therefore, as shown in Fig. 1, for  $m_{h^{\pm}}$  and  $m_{k^{\pm\pm}}$  which are respectively in the ranges 0 – 350 GeV, the transition strength is in the range  $1 \leq S < 2.4$ .

We see that the larger mass of  $h^{\pm}$  and  $k^{\pm\pm}$ , the larger cubic term ( $E$ ) in the effective potential but the strength of phase transition cannot be strong. Because, the value of  $\lambda$  also increases, so there is a tension between  $E$  and  $\lambda$  to make the first order phase transition. Also, when the masses of charged Higgses are too large,  $T_0, \lambda$  will be unknown or  $S \rightarrow \infty$ .



**Fig. 1.** When the solid contour of  $s = 2E/\lambda_{T_c} = 1$ , the dashed contour:  $2E/\lambda_{T_c} = 1.5$ , the dotted contour:  $2E/\lambda_{T_c} = 2$ , the dotted-dashed contour:  $2E/\lambda_{T_c} = 2.4$ , even and no-smooth contours:  $s \rightarrow \infty$

#### 4 SPHALERON RATE IN THE ZEE-BABU MODEL

When temperature drops below  $T_1$ , the effective potential appears a non-zero VEV. The transition from a zero vacuum to a non-zero vacuum through two ways. The first way which cross-over a barrier, was called sphaleron. The second way is quantum tunnelling, was called instanton.

The  $B$  violation can be seen through lepton number violation. Lepton violation can be read through Chern-Simon number or Winding number [4]. But we obtain the kinetic processes of Higgs field in EWPT which can be described by the transition rate between two VEVs. This rate is sphaleron rate. So sphaleron rate or Chern-Simon number is different from zero, driving  $\Delta B$  will not be zero [4].

**In order to calculate the sphaleron rate which have the contribution of the heavy particles** (their masses are larger than or equal  $W^\pm$  boson mass). Therefore, studying the sphaleron processes in the ZB model, we begin from the Lagrangian of the gauge-Higgs system:

$$\mathcal{L}_{\text{gauge-Higgs}} = -\frac{1}{4}F_{\mu\nu}^a F^{a\mu\nu} + (\mathcal{D}_\mu \phi)^\dagger (\mathcal{D}^\mu \phi) - V(\phi). \quad (11)$$

From Eq. (11), the energy functional which is the sum of the kinetic and potential constituents was taken over on space in the temporal gauge, will be the following form

$$\mathcal{E} = \int d^3\mathbf{x} \left[ (\mathcal{D}_\mu \phi)^\dagger (\mathcal{D}^\mu \phi) + V(\phi) \right], \quad (12)$$

here we assume that the least energy has the pure-gauge configurations, hence  $F_{ij}^a = 0$ .

We assume that the EWPT processes occur in the form of nucleation bubbles, and by using the temperature expansion of the effective potential at one loop, from the section 2, we can rewrite the energy functional in the spherical coordinate system as the form,

$$\mathcal{E} = 4\pi \int_0^\infty d^3x \left[ \frac{1}{2} \left( \nabla^2 v_\rho \right)^2 + V_{\text{eff}}(v_\rho; T) \right]. \quad (13)$$

Using the static field approximation, ie., VEV variable do not change in time, as follows,

$$\frac{\partial v_\rho}{\partial t} = 0, \quad (14)$$

we obtain

$$\mathcal{E} = \int d^3x \left[ \frac{1}{2} (\partial_i v_\rho)^2 + V_{\text{eff}}(v_\rho; T) \right]. \quad (15)$$

From the Lagrangian (11), we have the equation of motion for the VEV  $v_\rho$ :

$$\ddot{v}_\rho + \nabla^2 v_\rho - \frac{\partial V_{\text{eff}}(v_\rho, T)}{\partial v_\rho} = 0. \quad (16)$$

When VEV do not change in time as the condition (14), we can rewrite Eq. (16) in spherical coordinates:

$$\frac{d^2 v_\rho}{dr^2} + \frac{2}{r} \frac{dv_\rho}{dr} - \frac{\partial V_{\text{eff}}(v_\rho, T)}{\partial v_\rho} = 0. \quad (17)$$

The last, from Eq. (13) and (17), the sphaleron energy in the EWPT process,  $\mathcal{E}_{\text{sph. su}(2)}$ , has the following form:

$$\mathcal{E}_{\text{sph}} = 4\pi \int \left[ \frac{1}{2} \left( \frac{dv_\rho}{dr} \right)^2 + V_{\text{eff}}(v_\rho, T) \right] r^2 dr. \quad (18)$$

**We consider the electroweak phase transition as the formation of bubble nucleation.** In order to calculate the energy, we must solve the equation of motion (17) for the VEV of the Higgs field and obtain  $v_\rho(r)$ . Eq. (17) does not have an exact solution because the factor  $\frac{2}{r} \frac{dv_\rho}{dr}$  oscillates very strongly at  $r \rightarrow 0$ . Therefore, we should use approximations in the next subsections. The sphaleron rate per unit time, per unit volume,  $\Gamma/V$ , is characterized by a Boltzmann factor,  $\exp(-\mathcal{E}/T)$ , as follows [4, 20, 21]:

$$\Gamma/V = \kappa_{\text{brok}} \alpha^4 T^4 \exp(-\mathcal{E}/T), \quad (19)$$

where  $V$  is the volume of universe,  $T$  is the temperature,  $\mathcal{E}$  is the sphaleron energy, and  $\alpha = 1/30$ .  $\kappa_{brook}$  specifies the strength of EWPT. In this model, the strength is about 1, so  $\kappa_{brook} \simeq 1$ . We will compare the sphaleron rate with the Hubble constant, which describes the cosmological expansion rate at the temperature  $T$  [26,25]:

$$H^2 = \frac{\pi^2 g T^4}{90 M_{pl}^2}, \quad (20)$$

where  $g = 106.75$ ,  $M_{pl} = 2.43 \times 10^{18}$  GeV.

In order to have B violation, the sphaleron rate must be larger than the Hubble rate at the temperatures above the critical temperature (otherwise, B violation will become negligible during the Universe's expansion); however, the sphaleron process must be decoupled after the EWPT to ensure the generated BAU is not washed out [22].

#### 4.1 An upper bound of the sphaleron rates

The VEV of the Higgs field cannot be equal at every point in space, this can be seen in the equation of motion (17). So we suppose that the VEV of the Higgs fields dose not change from point to point. Due to this supposition, we have  $\frac{dv_x}{dr} = \frac{dv_\rho}{dr} = 0$ . Hence, from Eq. (17) we obtain

$$\frac{\partial V_{eff}(v_\rho)}{\partial v_\rho} = 0. \quad (21)$$

Equation (21) shows that  $v_\rho$  are the extremes of the effective potential. Therefore, the sphaleron energy (18) can be rewritten as

$$\mathcal{E}_{sph} = 4\pi \int V_{eff}(v_\rho, T) r^2 dr = \frac{4\pi r^3}{3} V_{eff}(v_\rho, T) \Big|_{v_{\rho_m}}, \quad (22)$$

where  $v_{\rho_m}$  is the VEV at the maximum of the effective potentials. From Eq. (22), the sphaleron energy is equal to the maximum heights of the potential barriers.

The Universe's volume at a temperature  $T$ , after the inflation and re-heating epoch, is given by  $V = \frac{4\pi r^3}{3} \sim \frac{1}{T^3}$ . Because the whole Universe is an identically thermal bath, the sphaleron energies are approximately

$$\mathcal{E}_{sph} \sim \frac{E^4 T}{4\lambda_T^3}. \quad (23)$$

From the definition (19), the sphaleron rates take the form

$$\Gamma \sim \alpha_w^4 T \exp\left(-\frac{E^4 T}{4\lambda_T^3}\right). \quad (24)$$

For the heavy particles,  $E, \lambda$  are constants. Hence, the sphaleron rate in this approximation is the linear functions of temperature. From Eq. (24), we estimate the value of the sphaleron rates as follows:

$$\Gamma \sim 10^{-4} \gg H \sim 10^{-13}. \quad (25)$$

This value is very large, so we assume it as an upper bound of the sphaleron rate. Therefore, sphaleron decoupling condition cannot be satisfied. For instance, as the temperature drops below the phase-transition temperature  $T_c$  and the Universe switches to the symmetry-breaking phase, the sphaleron rate is still much larger than the Hubble constant, and this makes the B violation washed out. By this consequence, the sphaleron process cannot occur identically in large regions of space; it can only take place in the microscopic regions or at each point in space.

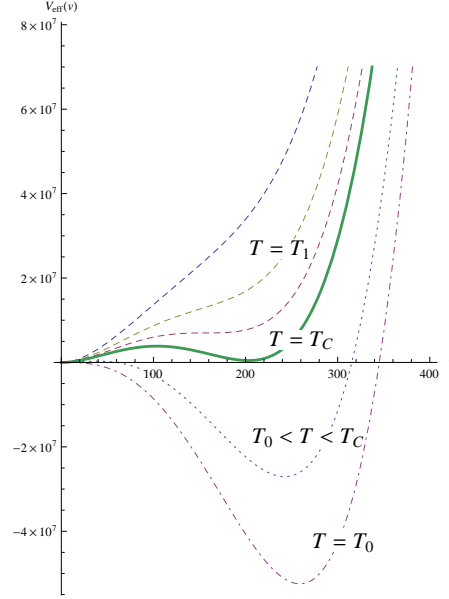
#### 4.2 Sphaleron rate in a thin-wall approximation

At every point in the early Universe, the effective potential varies as a function of VEV of the Higgs field and temperature, as illustrated in Fig. 2. If the temperature at a spatial location is higher than the bubble nucleation temperature  $T_1$ , then  $V_{eff}(v_\rho)$  at this location has only one the zero minimum, and this location is in a symmetric phase region.

As the temperature goes below  $T_1$ , the second minimum of  $V_{eff}(v_\rho)$  gradually forms, and a potential barrier which separates two minima gradually appears. The VEV can be transformed by thermal fluctuations. The phase transition

occurs microscopically, resulting in a tiny bubble of broken phase where the Higgs field  $\phi$  gets a nonzero expectation value.

As the temperature goes to  $T_c$ ,  $V_{eff}(v_\rho)$  at two minimums are equal together. But when the temperature goes below  $T_c$ , the second minimum becomes the lower one corresponding to a true vacuum, while the first minimum becomes the false vacuum. Such tiny true-vacuum bubbles at various locations in the Universe can occur randomly and expand in the midst of false vacuum. If the sphaleron rate is larger than the Universe's expansion rate, the bubbles can collide and merge until the true vacuum fills all space. However, if the sphaleron decoupling condition is satisfied after the transition, the sphaleron rate must be smaller than the cosmological expansion rate when the temperature goes from  $T_c$  to  $T_0$ , at which the first minimum of  $V_{eff}(v_\rho)$  completely disappears.



**Fig. 2.** The dependence of the effective potential on temperature

In a bubble of the EWPT, we have an the following approximation:

$$\frac{\partial V_{eff}(v_\rho)}{\partial v_\rho} \approx \frac{\Delta V_{eff}(v_\rho)}{\Delta v_\rho} = const \equiv M, \quad (26)$$

here  $\Delta v_\rho = v_{\rho_c}$ ,  $\Delta V_{eff}(v_\rho) = V_{eff}(v_{\rho_c}) - V_{eff}(0)$ , and  $v_{\rho_c}$  is a second minimum of the effective potential for the phase transition.

Now, we solve the equations of motion (17) for the VEV  $v_\rho$  by the approximation (26). Rewriting Eq. (17) in this approximation, we have

$$\frac{d^2 v_\rho}{dr^2} + \frac{2}{r} \frac{dv_\rho}{dr} = M. \quad (27)$$

In the cases that  $r \rightarrow \infty$  (the spatial locations are in the symmetric phase) or  $r \rightarrow 0$  (the spatial locations are in the broken phase), the VEVs must satisfy the boundary conditions:

$$\lim_{r \rightarrow \infty} v_\rho(r) = 0; \quad \left. \frac{dv_\rho(r)}{dr} \right|_{r=0} = 0. \quad (28)$$

In the bubble walls, the solutions of Eq. (27) take the form

$$v_\rho = \frac{M}{6} r^2 - A/r + B, \quad (29)$$

where  $A, B$  are the parameters to be specified.

The continuity of the scalar fields in the bubble results in the two following systems of equations:

$$\begin{cases} \frac{M}{6} R_b^2 - A/R_b + B = v_{\rho_c}, \\ \frac{M}{6} (R_b + \Delta l)^2 - A/(R_b + \Delta l) + B = 0, \end{cases} \quad (30)$$

**Table 1.** Sphaleron rate with  $m_{h^\pm} = 110$  GeV;  $m_{k^{\pm\pm}} = 200$  GeV

T[GeV]	$R[10^{-4} \times \text{GeV}^{-1}]$	$R/\Delta l$	$\varepsilon[\text{GeV}]$	$\Gamma[10^{-12} \times \text{GeV}]$	$H[10^{-14} \times \text{GeV}]$	$\Gamma/H$
154( $\sim T_1$ )	10	10	170.875	62694300	3.46826	$1.8 \times 10^9$
153	11	11	200.329	50999600	3.42278	$1.49 \times 10^9$
150.64 ( $T_c$ )	12	12	3418.22	0.029	3.1	$\sim 1$
140	15	15	6095.05	$2.13873 \times 10^{-11}$	2.86585	$7.46281 \times 10^{-10}$
130.35 ( $T_0$ )	20	20	5697.207	$1.67873 \times 10^{-11}$	2.4844	$6.757 \times 10^{-10}$

where  $R_b$  and  $\Delta l$  are, respectively, the radius and the wall thickness of a bubble nucleated.

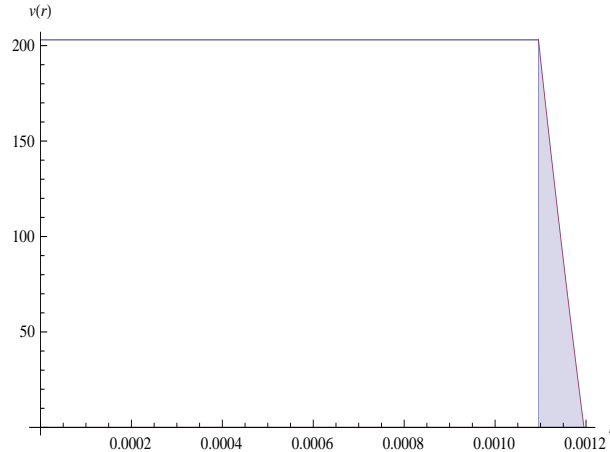
Solving the systems of Eq. (30), we obtain the solutions  $v_\rho$ , which are of the forms

$$v_\rho(r) = \begin{cases} v_{\rho_c}; & \text{when } r \leq R_b, \\ \frac{M}{6}r^2 - A/r + B; & \text{when } R_b < r \leq R_{b.su(2)} + \Delta l, \\ 0; & \text{when } R_b + \Delta l < r. \end{cases} \quad (31)$$

The system (30) has four equations, so we have to specify four unknown parameters [ $A$ ,  $B$ ,  $\Delta l$ , and  $R_b$ ]. Therefore the decoupling condition, the sphaleron rate is equal to the Hubble rate at  $T_c$ , has been used. This supposition relies on the requirement for avoiding the washout of the generated BAU after a phase transition, by which the sphaleron rate must be larger than the Hubble rate at temperatures above  $T_c$ , but the sphaleron rate must be smaller than the Hubble rate at temperatures below  $T_c$ .

The masses of heavy particles ( $h^\pm, k^{\pm\pm}$ ) are unknown so far. However, we can estimate their mass regions which satisfy the first-order phase transition conditions, and we choose any values in these regions for calculating the sphaleron energy. Although the strengths of the first-order EWPT are sufficiently strong ( $> 1$ ), they are not so strong ( $< 2.4$ ), hence the coefficients ( $\lambda_T, E, D$ ) in the effective potential are not meaningfully different for the different values in these regions. Here, **we choice**  $m_{h^\pm} = 110$  GeV;  $m_{k^{\pm\pm}} = 200$  GeV (**our choices are random**).

In Fig. 3, our respective solution  $v_\rho(r)$  are not as smooth as those in Refs. [22, 21, 23]. Because in this work, the bubble walls were been considered very thin, ie.,  $\Delta l \ll 1/T$  (while in Ref. [23], for instance,  $\Delta l \gg 1/T$ ). Inside the thin walls of bubbles,  $\frac{dv_\rho}{dr}$  is very large; this allows the Higgs field  $\phi$  to change their values over potential barriers. Therefore, the thinner the bubble walls, the larger the sphaleron rates.



**Fig. 3.** The solutions  $v(r)$  with  $m_{h^\pm} = 110$  GeV;  $m_{k^{\pm\pm}} = 200$  GeV. The regions in grey portray the thin walls of vacuum bubbles nucleated in each phase transition.

The behavior of the sphaleron rate is estimated in Table 1, in the cosmological expansion as the Universe cools through the respective critical temperature.

From Table 1, the broken phase of the EWPT starts at the bubble nucleation temperature  $T_1 \approx 154$  GeV in the bubbles with radius  $10 \times 10^{-4} \text{ GeV}^{-1}$ . At  $T_1$ , the sphaleron rate can be  $6.27 \times 10^{-5} \text{ GeV}$ , which is  $1.8 \times 10^9$  times larger than the Hubble rate ( $H = 3.1 \times 10^{-14} \text{ GeV}$ ). As the temperature drops below  $T_1$ , the sphaleron rate is larger than the Hubble rate and this lasts until the temperature reaches the critical temperature  $T_c = 150.64$  GeV. As the temperature goes from  $T_c$  to  $T_0$ , the sphaleron rate is smaller than the Hubble rate, and becomes negligible at



$T_0 = 130.35$  GeV when the transition ends. In addition, when the temperature is nearby  $T_C$ , the change of sphaleron rate is very strong.

## 5 Constraints on coupling factors in the Higgs potential

In order to have the first order phase transition,  $m_{h^\pm}$  and  $m_{k^{\pm\pm}}$  must be smaller than 350 GeV. Therefore, we obtain

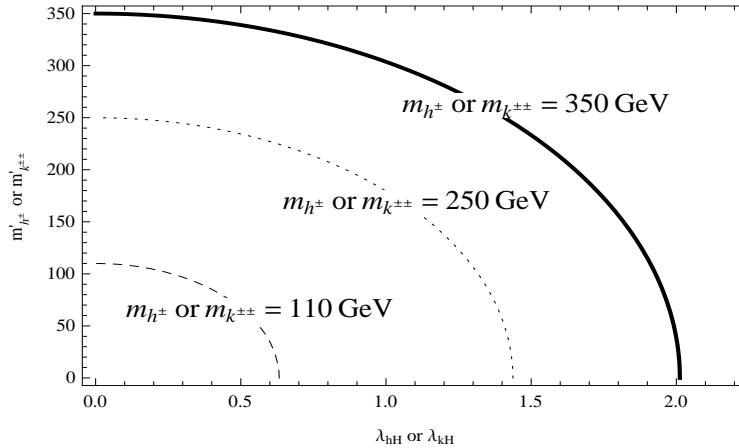
$$\frac{\lambda_{hH} v_0^2}{2} + m_{h'}^2 < (350 \text{ GeV})^2, \quad (32)$$

and

$$\frac{\lambda_{kH} v_0^2}{2} + m_k^2 < (350 \text{ GeV})^2. \quad (33)$$

In the ZB model, the tiny masses of neutrino are generated at two loops, so  $m_{h^\pm}$  and  $m_{k^{\pm\pm}}$  cannot be very heavy [18]. From the experimental point of view it is interesting to consider new scalars light enough to be produced at the LHC, theoretical arguments introduce that the scalar masses should be a few TeVs, to avoid unnaturally large one-loop corrections to the Higgs mass which would introduce a hierarchy problem. Therefore, these upper bounds of new scalar masses can be 2 TeVs [19]. Contacting to neutrino oscillation data, in the decay  $k^{\pm\pm} \rightarrow ll$ , the branching ratio to  $\tau\tau$  is very small in the ZB model, less than about 1%. Then, a conservative limit is  $m_{k^{\pm\pm}} > 200$  GeV. In the ZB model, we can have the decay  $k^{\pm\pm} \rightarrow h^\pm h^\pm$ , so  $2m_{h^\pm} < m_{k^{\pm\pm}}$ . Therefore, our results (Eqs. (32), (33)) are consistent with the above estimates.

These range values of  $m_{h^\pm}$  or  $m_{k^{\pm\pm}}$  are shown in Fig.4. We obtain  $0 < \lambda_{hH} < 2$  and  $0 < \lambda_{kH} < 2$ . However, we need to have other considerations in order to find these accurate values of  $m_{h^\pm}$  and  $m_{k^{\pm\pm}}$ .



**Fig. 4.** Solid contour:  $m_{h^\pm}/m_{k^{\pm\pm}} = 350$  GeV, dotted contour:  $m_{h^\pm}/m_{k^{\pm\pm}} = 250$  GeV; dashed contour:  $m_{h^\pm}/m_{k^{\pm\pm}} = 110$  GeV

Recently, the experimental groups at LHC (ATLAS and CMS Collaborations) [28] have reported an experimental anomaly in diboson production with apparent excess in boosted jets of the  $W^+W^-$ ,  $W^\pm Z$  and  $ZZ$  channels at around 2 TeV invariant mass of the boson pair. To deal with the LHC data, one need to introduce some multiply charged bosons with mass of TeV scale (for more details, the reader is referred to [29]). In addition the calculation the Higgs coupling to photons (due to charged particles in the loop diagram) can be related to neutrino mass and CP violation which are the key of matter and antimatter asymmetry. This study will be investigated in a future publication.

## 6 CONCLUSION AND OUTLOOKS

We have investigated the EWPT and sphaleron rate in the ZB model using the high-temperature effective potential. The EWPT is strengthened by the new scalars to be the strongly first-order, the phase transition strength is in the range 1 – 2.4. The sphaleron rate satisfies the decoupling condition.  $h^\pm, k^{\pm\pm}$  are triggers for the first-order EWPT. Our results may be further than the results in Ref. [27].

In the ZB model, the tiny mass of neutrino which can be explain in two loops interactions of charged Higgs with neutrino, can be an reason of the matter-antimatter asymmetry and CP-violation. The behavior of charged Higgs is also very interested. Therefore, in the next works, we can be investigated coupling contacts  $m'_{h^\pm/k^\pm\pm}$  by using neutrino data. We will investigate the CP- violation and beyond issues of the baryon asymmetry prolem through neutrino physics.

With this region of self couplings in the Higgs potential, we can serve as basis for the calculation of cross section of the decay Higgs to photons and evaluate and extend the Zee-Babu model when connected to the data of LHC.

## ACKNOWLEDGMENTS

This research is funded by Vietnam National Foundation for Science and Technology Development (NAFOSTED) under grant number 103.01-2014.51 and by Ton Duc Thang university.

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